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A NOTE ON HEAT-TRANSFER MECHANISM AS APPLIED TO FLOWING GRANULAR MEDIA

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NOMENCLATURE

d,	particle diameter [m];
Fo_m,	$\alpha\tau/d^2$ average Fourier number;
$\bar{h}$,	average film heat-transfer coefficient [W/m^2 K];
k,	effective thermal conductivity of granular medium [W/mK];
k_g,	gas thermal conductivity [W/mK];
K,	surface conductance [W/m^2 K];
L,	length of plate [m];
Nu_c,	Kd/k_g, contact Nusselt number;
$\overline{Nu_d^*}$,	$\bar{h}d/k_g$, average Nusselt number;
$\overline{Nu_L}$,	$\bar{h}L/k$, average Nusselt number;
Nu_m,	$\bar{h}d/k$, average Nusselt number;
Pe_L,	UL/α, Peclet number;
Pe_L^*,	$Pe_L(k/k_g)^2(d/L)^2$, Peclet number;
U,	velocity of moving granular medium [m/s].

Greek symbols

α,	effective thermal diffusivity of granular medium [m^2/s];
γ,	experimental constant;
τ,	L/U, mean packet residence time [s];
χ,	experimental constant.

INTRODUCTION

OVER the last twenty years there has been a continued interest in the mechanism of heat transfer between granular media and surfaces, as in dense-phase conveying, or in fluidized beds. Several mechanisms of heat transfer have been considered, however, two main approaches can be discerned; the packed-bed approach as used by Mickley and Fairbanks [1] and Baskakov [2] for example; and the single particle approach as used by Botterill and Williams [3] and Ziegler and Agrawal [4]. The recent paper by Sullivan and Sabersky [5] proposed two models describing heat transfer in flowing granular media; the former similar to that of Gabor [6]; the latter identical to that of Baskakov [7] (a modified Mickley–Fairbanks approach). As the interesting paper by Sullivan and Sabersky discounted the wide literature of the fluidized bed field, it is worthwhile and instructive to consider and analyze their data and conclusions in the light of fluidized bed heat-transfer knowledge.

THEORETICAL CONSIDERATIONS

One of the traditional approaches to heat transfer in moving granular media has been to solve the transient heat-transfer equations during the time of contact, or the mean residence time of the medium on the surface, τ, which can be readily derived from the variables used by Sullivan and Sabersky by transforming

$$\tau = L/U.$$

The groups most commonly used in transient conduction studies can then readily be related to those used by Sullivan and Sabersky, as shown in Table 1.

Table 1. Transformation of variables

Usual variable		Sullivan and Sabersky [5] form
Fo_m	$\dfrac{\alpha\tau}{d^2}$	$(Pe_L)^{-1}\left(\dfrac{L}{d}\right)^2$
		$(Pe_L^*)^{-1}\left(\dfrac{k}{k_g}\right)^2$
Nu_m	$\dfrac{\bar{h}d}{k}$	$\overline{Nu_L}\,\dfrac{d}{L}$
		$\overline{Nu_d^*}\,\dfrac{k_g}{k}$
Nu_c	$\dfrac{dK}{k_g}$	$1/\chi$

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It is particularly instructive to consider their [5] equations (12) and (14), as the contact resistance (or contact conductance) concept is of fundamental importance in granular media heat-transfer problems:

Equation (12) transforms to

$$Nu_m = \frac{1}{\gamma(L/d) + \sqrt{\left(\frac{\pi}{4} Fo_m\right)}} \tag{1}$$

and equation (14) transforms to

$$Nu_m = \frac{1}{\chi(k/k_g) + \sqrt{\left(\frac{\pi}{4} Fo_m\right)}}. \tag{2}$$

The curve-fitting values for γ in the paper appear to be very good; that for χ is very dubious (due to the data smoothing). However, equation (1) and (2) show that,

$$\chi = \gamma\left(\frac{L}{d}\right)\cdot\left(\frac{k_g}{k}\right). \tag{3}$$

Table 2. Retabulation of data

Particles used	d[μm]	L/d	k/k_g	γ	χ
Trafic beads	330	46·15	7·8	0·022	0·130
		76·92	7·8	0·013	0·128
Glass beads	1346	11·32	7·2	0·058	0·091
		18·87	7·2	0·035	0·092
Sand	203	75	13·1	0·042	0·241
		125	13·1	0·025	0·239
Mustard seeds	2159	7·06	5·23	0·062	0·084
		11·76	5·23	0·037	0·083

From equation (3) and the data in their paper [5] Table 2 is formed. As expected, χ is not a constant, but depends on the shape of the granules as pointed out by the authors. The quantity χ, which one may imagine to represent an equivalent stationary gas film, should therefore never be taken as a universal constant. However, means are available to predict the heat transfer condition over the surface and these will be discussed below.

THE CONTACT RESISTANCE CONCEPT

The use of a contact resistance as a technique to match the predictions with practical results is very common in granular medium heat-transfer theory, [3, 7]. This contact resistance is explained physically in terms of the complicated packing and flow observed near surfaces and is usually equated with a stagnant film at the wall whose properties are defined in terms of a thickness (usually a fraction of a particle diameter) and a conductivity. This conductivity may be that of the gas [3] or based upon a low voidage packed bed at the wall [7]. There is no physical reason to believe that a stagnant film exists so that the prediction of χ, or more usually Nu_c, in terms of a film can only be considered as being a mathematical expedient which allows for the complicated variation of thermophysical and geometrical properties near a surface.

It is apparent that a full understanding of heat-transfer mechanism in granular media is critically dependant upon an adequate description of the zone within one particle diameter of the heat transfer surface being developed, as in this region the use of mean packet-based properties cannot be justified. A numerical solution of the transient heat diffusion equation allowing for the non-homogeneous nature of the thermophysical properties close to the surface has been developed recently [8], in which a particular voidage field close to a surface is used to generate the variation of thermophysical properties, in order that no empirically derived properties need be used in calculating heat-transfer coefficients. This approach requires the same inputs concerning packed bed thermal conductivity, residence time and particle size as the conventional theory and also requires a reasonable approximation to the voidage field near the surface but should allow accurate predictions in those cases where a value for the contact resistance has not been evaluated. It has the disadvantage of being numerical, however, through investigating the region close to the surface an improved understanding of the heat transfer process should prove possible.

The use of a contact resistance should be looked upon only as an *a-posteriori* curve-fitting technique for data correlation or for design and is a valuable tool for empirical studies.

CONCLUSIONS

1. The paper by Sullivan and Sabersky [5] can be reinterpreted in terms of the variables commonly used in granular medium heat transfer papers and some of tne similarities are demonstrated.
2. The use of a contact resistance based on a constant χ is shown to be a source of errors due to the wide variability with geometry etc. The reasons for these limitations and a method of overcoming them are discussed.

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